

Suppose N is the real number of live tuples, and P_0, P_1 are the old and the new numbers of pages respectively. We estimate the new tuple density using the old density $\rho_0 = N/P_0$ and the density reported by ANALYZE, ρ_A . In the approximation that ANALYZE is ideal and returns the current real tuple density, $\rho_A = \rho_1 = N/P_1$. Therefore, the new tuple density can be estimated as

$$\begin{aligned}\rho_1^{est} &= (1 - \alpha)\rho_0 + \alpha\rho_A \\ &= (1 - \alpha)\frac{N}{P_0} + \alpha\frac{N}{P_1},\end{aligned}$$

where $\alpha < 1$ is the moving average factor. Then, we estimate the new number of tuples using the density we computed,

$$\begin{aligned}N^{est} &= \rho_1^{est} P_1 \\ &= (1 - \alpha)\frac{N}{P_0}P_1 + \alpha\frac{N}{P_1}P_1 \\ &= (1 - \alpha)N\frac{P_1}{P_0} + \alpha N.\end{aligned}$$

The change in the number of tuples is given by

$$\begin{aligned}\Delta N &= N^{est} - N \\ &= (1 - \alpha)N\frac{P_1}{P_0} + \alpha N - N \\ &= (1 - \alpha)N\left(\frac{P_1}{P_0} - 1\right).\end{aligned}$$

Given that $P_1 > P_0$, and other terms of ΔN are positive as well, we can conclude that $\Delta N > 0$.